# INSTABILITY OF A STATIONARY THIN LAYER OF A MAGNETIZABLE LIQUID

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§1. The prospects of using thin films and film flows of a magnetizable liquid in chemical-engineering heat- and mass-transfer devices necessitate an investigation of their stability in various magnetic fields, especially since the investigations of this problem which have been made so far on liquids of infinite depth have shown a specific magnetic-instability mechanism of the free surface and broad possibilities of controlling the surface by a magnetic field [1-5]. A fundamental role in the equations derived is played by the minimum phase velocity of surface capillary-gravitational waves $u_m^2 = 2\sqrt{(\rho_1 - \rho_2)g\alpha}/(\rho_1 + \rho_2)$, which determines the stability limit, and the corresponding wave number $k_m = \sqrt{(\rho_1 - \rho_2)g/\alpha}$, which determines the wavelength of the surface perturbations after the loss of stability ($\rho_1$ and $\rho_2$ are the densities of the adjoining liquids; $\alpha$ is the surface tension between them; and g is the acceleration due to gravity).

For thin layers of liquid the stability conditions undergo serious changes if only because the phase velocity of the capillary-gravitational waves becomes different and the finite thickness of the layer of magnetizable liquid profoundly affects the distribution of magnetic-field perturbations inside and outside the layer. The problem is also directly related to the problem of the effect of the magnetic properties of the media adjacent to the layer of liquid on its stability.

§2. We consider the problem of the stability of a layer of magnetizable liquid when there is a tangential velocity discontinuity on its free surface. The geometry of the problem is shown in Fig. 1. A stationary layer of magnetizable liquid 1 of thickness h is bounded below ($y = -h$) by a plane solid surface 3, and above ($y = 0$) by a stream of another liquid 2 moving along the free surface with a velocity $\mathbf{u}_0$. The whole system is in the vertical gravitational field $\mathbf{g}$ and a uniform magnetic field $\mathbf{H}_0$ which has the components $H_{0x}$, $H_{0y}$, and $H_{0z}$ in the solid at infinity. The physical characteristics of the media and the unknown quantities in them have subscripts corresponding to the numbers of the media.

The liquids under consideration are assumed incompressible, inviscid, and electrically nonconducting, and all three media are assumed to obey the linear-magnetization law $\mathbf{M} = \chi\mathbf{H}$, where $\mathbf{M}$ is the magnetic moment per unit volume of the material; $\chi$ is its magnetic susceptibility; and $\mathbf{H}$ is the magnetic-field intensity.

Under the above assumptions the motion of the magnetizable liquid is described by the equations of ferrohydrodynamics [1-5]

$$\rho[\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v}] = -\nabla p + \rho\mathbf{g} + \mu_0 M\nabla H,$$
$$\operatorname{div}\mathbf{v} = 0,\ \operatorname{rot}\mathbf{H} = 0,\ \operatorname{div}\mathbf{B} = 0,\ \mathbf{B} = \mu_0(1+\chi)\mathbf{H} = \mu\mathbf{H}, \tag{2.1}$$

where $\mathbf{v}$ is the velocity; t is the time; p is the pressure; $\mu_0$ and $\mu$ are the magnetic permeabilities of free space and the medium; and $\mathbf{B}$ is the magnetic induction.

At all interfaces the tangential component of the magnetic-field intensity and the normal component of the magnetic induction must be continuous

$$\mathbf{n} \times [\mathbf{H}_i - \mathbf{H}_j] = 0,\ \mathbf{n}\cdot(\mathbf{B}_i - \mathbf{B}_j) = 0, \tag{2.2}$$

where $\mathbf{n}$ is a unit vector normal to the surface of separation

$$\mathbf{n} = \{-q\partial F/\partial x,\ q,\ -q\partial F/\partial z\},\ q \equiv [1 + (\partial F/\partial x)^2 + (\partial F/\partial z)^2]^{-1/2}, \tag{2.3}$$

and $y = F(x, z)$ is the equation of this surface. In addition, the normal stresses must be equal at the interface between the liquids

$$p_i - p_j = -\frac{1}{2}\mu_0[(\mathbf{M}_i\mathbf{n})^2 - (\mathbf{M}_j\mathbf{n})^2] + \alpha(R_1^{-1} + R_2^{-1}), \tag{2.4}$$

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on the solid boundary ($y = -h$) the normal component of the velocity must vanish ($v_{1y} = 0$), and on the free surface ($y = 0$)

$$\partial F/\partial t + \mathbf{v}_1\nabla F = v_{1y},\ \partial F/\partial t + \mathbf{v}_2\nabla F = v_{2y}, \tag{2.5}$$

where $R_1$ and $R_2$ are the principal radii of curvature of the surface. Except for the boundary conditions on the solid surface the statement of the problem is the same as in [1-3].

§3. The problem is solved in the linear approximation. Perturbations of the quantities being sought are denoted by primes and equilibrium values by a subscript 0.

From (2.1) the equations for the potentials of the perturbations of the velocity $\mathbf{v}' = -\nabla\varphi$ and the magnetic-field intensity $\mathbf{H}' = \nabla\Phi$ are

$$p' = -\rho g y' + \rho\frac{\partial\varphi}{\partial t} + \rho u_0\frac{\partial\varphi}{\partial x} - \mu_0 M_0\frac{\mathbf{H}_0\mathbf{H}'}{H_0},$$
$$\Delta\varphi = 0;\ \Delta\Phi = 0.$$

For perturbations which are periodic in the (x0z) plane the last two equations have the general solutions

$$\varphi = (D_- e^{-ky} + D_+ e^{ky})e^{i(\mathbf{kr}-\omega t)},$$
$$\Phi = (c_- e^{-ky} + c_+ e^{ky})e^{i(\mathbf{kr}-\omega t)},\ \mathbf{k} = [k_x,\ 0,\ k_z] \tag{3.1}$$

($\mathbf{k}$ is the wave number; $\omega$ is the frequency; $\mathbf{r} = [x, y, z]$), which by taking account of the necessary fall-off at infinity and the boundary conditions on the solid surface ($y = -h$) give for each of the media

$$\varphi_1 = D_1 \operatorname{ch} k(y+h)e^{i(\mathbf{kr}-\omega t)},\ \varphi_2 = D_2 e^{-ky}e^{i(\mathbf{kr}-\omega t)}),$$
$$\Phi_1 = c_3 e^{-kh}\ [\operatorname{ch} k(y+h) + (\mu_3/\mu_1)\operatorname{sh} k(y+h)]e^{i(\mathbf{kr}-\omega t)},$$
$$\Phi_2 = c_2 e^{-ky}\, e^{i(\mathbf{kr}-\omega t)},\ \Phi_3 = c_3 e^{ky}\, e^{i(\mathbf{kr}-\omega t)},$$
$$F = Ae^{i(\mathbf{kr}-\omega t)},\ H_{0x} = H_{0x1},\ H_{0z} = H_{0z1},\ (1+\chi_3)H_{0y} = (1+\chi_1)H_{0y1}.$$

In the linear approximation the boundary conditions (2.2)-(2.5) on the free surface of separation F take the form

$$\frac{\partial\Phi_1}{\partial x} - \frac{\partial\Phi_2}{\partial x} + \frac{(1+\chi_3)(\chi_2-\chi_1)}{(1+\chi_1)(1+\chi_2)}H_{0y}\frac{\partial F}{\partial x} = 0,$$
$$\frac{\partial\Phi_1}{\partial z} - \frac{\partial\Phi_2}{\partial z} + \frac{(1+\chi_3)(\chi_2-\chi_1)}{(1+\chi_1)(1+\chi_2)}H_{0y}\frac{\partial F}{\partial z} = 0,$$
$$H_{0x}\frac{\partial F}{\partial x} + H_{0z}\frac{\partial F}{\partial z} + \frac{1+\chi_1}{\chi_2-\chi_1}\frac{\partial\Phi_1}{\partial y} - \frac{1+\chi_2}{\chi_2-\chi_1}\frac{\partial\Phi_2}{\partial y} = 0, \tag{3.2}$$
$$H_{0x1} = H_{0x2},\ H_{0z1} = H_{0z2},$$
$$(1+\chi_1)H_{0y1} = (1+\chi_2)H_{0y2};$$

$$-(\rho_1-\rho_2)gF + \rho_1\frac{\partial\varphi_1}{\partial t} - \rho_2\frac{\partial\varphi_2}{\partial t} - \rho_2 u_0\frac{\partial\varphi_2}{\partial x} +$$
$$+\alpha\left(\frac{\partial^2F}{\partial x^2} + \frac{\partial^2F}{\partial z^2}\right) + \mu_0 H_{0x}\left(\chi_1\frac{\partial\Phi_1}{\partial x} - \chi_2\frac{\partial\Phi_2}{\partial x}\right) +$$
$$+\mu_0 H_{0z}\left(\chi_1\frac{\partial\Phi_1}{\partial z} - \chi_2\frac{\partial\Phi_2}{\partial z}\right) - \mu_0\frac{1+\chi_3}{(1+\chi_1)(1+\chi_2)}H_{0y}\left[\chi_2(1+\chi_1)\frac{\partial\Phi_1}{\partial y} -\right.$$
$$\left.-\chi_1(1+\chi_2)\frac{\partial\Phi_2}{\partial y}\right] = 0; \tag{3.3}$$

$$\frac{\partial F}{\partial t} = -\frac{\partial\varphi_1}{\partial y},\ \frac{\partial F}{\partial t} + u_0\frac{\partial F}{\partial x} = -\frac{\partial\varphi_2}{\partial y}. \tag{3.4}$$

We substitute the solutions (3.1) into the boundary conditions (3.2)-(3.4) and assume that the amplitude of the surface perturbations is small in comparison with their wavelength, which enables us to replace the boundary conditions on the surface F by the boundary conditions on the plane surface $y = 0$ after performing all the differentiation operations. This gives the dispersion equation for surface waves in the form

$$(\rho_1-\rho_2)g + \alpha k^2 - \frac{\rho_1\omega^2}{k\operatorname{th}kh} - \frac{\rho_2(\omega-k_x u_0)^2}{k} +$$
$$+\frac{(\mu_2-\mu_1)^2}{\mu_2}\frac{(\mathbf{kH}_0)^2}{k}\frac{1+(\mu_3/\mu_1)\operatorname{th}kh}{1+\mu_3/\mu_2+(\mu_1/\mu_2+\mu_3/\mu_1)\operatorname{th}kh} -$$

$$-\frac{\mu_3^2(\mu_2-\mu_1)^2}{\mu_1\mu_2^2}H_{0y}^2 k\frac{\mu_3/\mu_1+\operatorname{th}kh}{1+\mu_3/\mu_2+(\mu_1/\mu_2+\mu_3/\mu_1)\operatorname{th}kh}=0,$$

from which the phase velocity of the wave $v = \omega/k$ is

$$v=\frac{\rho_2 u_0\cos\psi}{\rho_1/\operatorname{th}kh+\rho_2}\pm\left\{\frac{(\rho_1-\rho_2)g+\alpha k^2}{k(\rho_1/\operatorname{th}kh+\rho_2)}-\right.$$
$$-\frac{\rho_1\rho_2 u_0^2\cos^2\psi}{(\rho_1/\operatorname{th}kh+\rho_2)^2\operatorname{th}kh}-\frac{\mu_3(\mu_2-\mu_1)^2}{\mu_1\mu_2^2}H_{0y}^2\frac{1}{(\rho_1/\operatorname{th}kh+\rho_2)}\times \tag{3.5}$$
$$\times\frac{\mu_3/\mu_1+\operatorname{th}kh}{1+\mu_3/\mu_2+(\mu_1/\mu_2+\mu_3/\mu_1)\operatorname{th}kh}+\frac{(\mu_2-\mu_1)^2}{\mu_2}H_{0\tau}^2\times$$
$$\left.\times\frac{1}{(\rho_1/\operatorname{th}kh+\rho_2)}\frac{1+(\mu_3/\mu_1)\operatorname{th}kh}{1+\mu_3/\mu_2+(\mu_1/\mu_2+\mu_3/\mu_1)\operatorname{th}kh}\cos^2(\psi-\sigma)\right\}^{1/2},$$

where $\psi$ is the angle between the vectors $\mathbf{k}$ and $\mathbf{u}_0$; and $\sigma$ is the angle between the tangential component of the magnetic-field intensity $\mathbf{H}_{0\tau} = [H_{0x}, 0, H_{0z}]$ and $\mathbf{u}_0$.

The change in sign of the radical from + to — in the expression for the phase velocity indicates the onset of instability of the free surface. The remainder of the present article is devoted to an analysis of Eq. (3.5) from this point of view.

§4. First let us discuss some of the features of the surface instability on an infinite layer ($kh \gg 1$) of a magnetizable liquid which were not noted in the papers cited above. If there is no discontinuity in the tangential velocity at the surface ($u_0 = 0$), the instability condition has the form

$$\frac{(\mu_1-\mu_2)^2\mu_1}{(\mu_1+\mu_2)\mu_2}H_{0y}^2-\frac{(\mu_1-\mu_2)^2}{\mu_1+\mu_2}H_{0\tau}^2\cos^2\theta>\frac{(\rho_1-\rho_2)g+\alpha k^2}{k}$$

($\theta = \psi - \sigma$ is the angle between the vectors $\mathbf{k}$ and $\mathbf{H}_{0\tau}$), from which it follows that for $\theta = \pi/2$ instability begins at the same values of the normal component of the field as for $\mathbf{H}_{0\tau} = 0$. Thus, in this situation the tangential magnetic field does not affect the stability of the free surface of the liquid, but does change its form significantly after the onset of instability. Instead of a structure which is periodic in two directions (cell), if there is no tangential field [1] the structure on the surface of the liquid will be periodic only in the direction perpendicular to the tangential component of the field (ridges with their axes parallel to the field).

Some peculiarities occur also in the effect of the tangential magnetic field on the stability of the tangential velocity discontinuity. For $H_{0y} = 0$ the condition for its stability is

$$\frac{\rho_1\rho_2}{\rho_1+\rho_2}u_0^2\cos^2\psi-\frac{(\mu_1-\mu_2)^2H_{0\tau}^2}{\mu_1+\mu_2}\cos^2(\psi-\sigma)>\frac{(\rho_1-\rho_2)g+\alpha k^2}{k}, \tag{4.1}$$

from which it follows that when the tangential magnetic field is perpendicular to the flow velocity ($\sigma = \pi/2$) it does not affect the stability of the discontinuity and its maximum stabilizing effect occurs when it is parallel to $\mathbf{u}_0$. For $\pi/2 > \sigma > 0$ the critical velocity $u_{0*}$ lies in the range

$$2\sqrt{(\rho_1-\rho_2)g\alpha}<\frac{\rho_1\rho_2}{\rho_1+\rho_2}u_{0*}^2<2\sqrt{(\rho_1-\rho_2)g\alpha}+\frac{(\mu_2-\mu_1)^2}{\mu_1+\mu_2}H_{0\tau}^2.$$

From (4.1) it also follows that for $\mathbf{g} = 0$, $\alpha = 0$, or $\rho_1 = \rho_2$ and $\mathbf{H}_{0\tau} \nparallel \mathbf{u}_0$, the tangential velocity discontinuity is absolutely unstable with respect to perturbations which propagate perpendicular to the field ($\psi - \sigma = \pi/2$) and have long wavelengths ($k \to 0$) for $\mathbf{g} = 0$ or $\rho_1 = \rho_2$ and have short wavelengths ($k \to \infty$) for $\alpha = 0$. In these cases

$$v=\frac{\rho_2 u_0\cos\psi}{\rho_1+\rho_2}\left[1\pm i\sqrt{\rho_1/\rho_2}\right].$$

Only in a field parallel to the flow velocity ($\sigma = 0$) is it possible to have a stable discontinuity under the condition [3]

$$\frac{\rho_1\rho_2}{\rho_1+\rho_2}u_0^2<\frac{(\mu_1-\mu_2)^2}{\mu_1+\mu_2}H_{0\tau}^2.$$

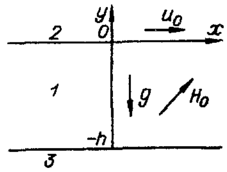


Fig. 1

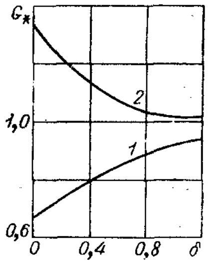


Fig. 2

Let us consider the effect of the magnetic properties of the bounding media and the finite thickness of the liquid layer on its stability. If there is no tangential velocity discontinuity ($u_0 = 0$), it follows from (3.5) that the most dangerous perturbations are those with a wave vector perpendicular to the tangential component of the field ($\psi - \sigma = \pi/2$) with respect to which the instability condition of the layer has the form

$$H_{0y}^2 > \frac{\mu_1\mu_2^2}{\mu_3(\mu_2-\mu_1)^2}\,\frac{1+\mu_3/\mu_2+(\mu_1/\mu_2+\mu_3/\mu_1)\,\mathrm{th}\,kh}{\mu_3/\mu_1+\mathrm{th}\,kh}\,\frac{(\rho_1-\rho_2)g+\alpha k^2}{k}. \tag{4.2}$$

It is convenient to minimize the right-hand side of this condition with respect to the dimensionless ratio of k to the critical wave number for an infinite layer $\sqrt{(\rho_1 - \rho_2)g/\alpha}$.

Introducing $S = k/\sqrt{(\rho_1 - \rho_2)g/\alpha}$ and the dimensionless number

$$G = \frac{\mu_3^2(\mu_2-\mu_1)^2}{(\mu_1+\mu_2)\mu_1\mu_2}\,\frac{H_{0y}^2}{2\sqrt{(\rho_1-\rho_2)g\alpha}},$$

which determines the onset of instability, we obtain the instability condition (4.2) in the form

$$G > \frac{1+S^2}{2S}\,\frac{1+\alpha_{32}+(\alpha_{12}+\alpha_{31})\,\mathrm{th}\,\delta S}{(1+\alpha_{12})(\alpha_{31}+\mathrm{th}\,\delta S)}, \tag{4.3}$$

where $\alpha_{ik} = \mu_i/\mu_k$; and $\delta = h\sqrt{(\rho_1 - \rho_2)g/\alpha}$ is the dimensionless thickness of the layer.

The critical values of the parameter G are determined by the minimum of the right-hand side of (4.3) with respect to S and correspond in the present case to a stationary character of the instability ($v = 0$). For an infinite layer ($\delta >> 1$) $G_* = 1$ and $S_* = 1$. In (4.3) the critical values of $G_*$ are determined by both the dimensionless thickness of the layer $\delta$ and the relative magnetic characteristics $\alpha_{ik}$ of the media bounding the layer.

For $\delta S << 1$ in the zero approximation it follows from (4.3) that

$$G_* = \frac{1+\alpha_{23}}{1+\alpha_{21}} = \frac{1+\mu_2/\mu_3}{1+\mu_2/\mu_1},\ S_* = 1. \tag{4.4}$$

The condition $\delta S << 1$ in this case is transformed into $\delta << 1$, i.e., $h << \sqrt{\alpha/(\rho_1 - \rho_2)g}$.

Analysis of Eq. (4.4) gives the following results: $G_* > 1$ for $\mu_1 > \mu_3$; $G_* < 1$ for $\mu_1 < \mu_3$. This shows that the presence of a solid boundary medium with a magnetic permeability larger than that of the liquid decreases the stability of the layer, and in the opposite case the stability of the layer is increased.

The dependence of $G_*$ on the thickness of the layer $\delta$ calculated by Eq. (4.3) is shown in Fig. 2 for two cases of different media bounding the layer of magnetizable liquid. In the first case (curve 1) $\alpha_{31} = 1000$, $\alpha_{12} = 2$, and $\alpha_{32} = 2000$, which corresponds to an infinitely large magnetic permeability of the solid (medium 3) bounding the layer, and a nonmagnetic liquid (medium 2) over the layer for a magnetic permeability of the layer $\mu_1 = 2\mu_0$; in the second case (curve 2) $\alpha_{31} = 0.5$, $\alpha_{12} = 2$, and $\alpha_{32} = 1$, which corresponds, for example, to a layer of magnetizable liquid with $\mu_1 = 2\mu_0$ bounded by nonmagnetic media. In the first case $G_*$ decreases with decreasing $\delta$, while in the second case it increases to values determined by Eq. (4.4). In both cases $G_*$ is practically unity for $\delta > 1$.

The critical values of the wave number in both cases vary less rapidly with $\delta$ than with $G_*$. In the first case they are decreased ($0.8 \leq S_* < 1$), in the second case they are increased ($1 < S_* \leq 1.2$), and in the limiting cases ($\delta < 0.1$ and $\delta > 1$) they are practically 1.

It follows from (4.4) that significant stabilization of a layer of magnetizable liquid can be achieved by bounding it above with a magnetizable liquid having a magnetic permeability much larger than that of the solid. In this case if $\mu_1 \simeq \mu_2 \gg \mu_3$, $G_* \sim \mu_2/\mu_3$; if $\mu_2 \gg \mu_3$ and $\mu_2 \gg \mu_1$, $G_* \sim \mu_1/\mu_3$.

Let us now consider the stabilizing effect of a magnetic field tangential to the surface ($H_{oy} = 0$) on the stability of the layer when there is a tangential velocity discontinuity ($u_0 \neq 0$). This stabilizing effect is maximum when the tangential field is parallel to the flow velocity ($\sigma = 0$). In this case the stability has a wave character and the instability condition has the form

$$u_0^2 > \frac{(\rho_1-\rho_2)g+\alpha k^2}{k}\,\frac{(\rho_1+\rho_2\,\mathrm{th}\,kh)}{\rho_1\rho_2}\left\{1+\frac{(\mu_1-\mu_2)^2H_{0\tau}^2k}{\mu_2\,[(\rho_1-\rho_2)g+\alpha k^2]}\times\right.$$
$$\left.\times\frac{1+\alpha_{31}\,\mathrm{th}\,kh}{1+\alpha_{32}+(\alpha_{12}+\alpha_{31})\,\mathrm{th}\,kh}\right\}. \qquad (4.5)$$

The critical values of $u_{o*}^2$ are determined by the minimum of the right-hand side of Eq. (4.5) with respect to k

for kh >> 1, $k_* = \sqrt{(\rho_1-\rho_2)g/\alpha}$,

$$u_{0*}^2 = \frac{2(\rho_1+\rho_2)}{\rho_1\rho_2}\sqrt{(\rho_1-\rho_2)g\alpha}\left[1+\frac{(\mu_1-\mu_2)^2H_{0\tau}^2}{\mu_2(1+\alpha_{12})\sqrt{(\rho_1-\rho_2)g\alpha}}\right]; \qquad (4.6)$$

for kh << 1, $k_* = \sqrt{(\rho_1-\rho_2)g/\alpha}$,

$$u_{0*}^2 = \frac{2}{\rho_2}\sqrt{(\rho_1-\rho_2)g\alpha}\left[1+\frac{(\mu_1-\mu_2)^2H_{0\tau}^2}{\mu_2(1+\alpha_{12})\sqrt{(\rho_1-\rho_2)g\alpha}}\,\frac{\mu_2+\mu_1}{\mu_2+\mu_3}\right]. \qquad (4.7)$$

The stabilizing action of the tangential magnetic field is expressed by the second term in the square bracket of Eqs. (4.6) and (4.7), from which it follows that in the two limiting cases of thick and thin layers of magnetizable liquid the critical wave length is the same. Bounding the layer by a solid with a magnetic permeability larger than that of the liquid ($\mu_3 > \mu_1$) decreases the stabilizing effect of the tangential magnetic field. For $\mu_1 > \mu_3$ it is increased.

Thus, generalizing the results of the problem considered, it can be concluded that a bounding solid with a magnetic permeability larger than that of the magnetizable liquid exerts a destabilizing effect on the stability of the liquid layer; on the other hand a bounding solid with a $\mu$ smaller than that of the liquid stabilizes the layer.

In conclusion we note that while it is possible to speak of the instability of the free surface of a thick layer of magnetizable liquid, these concepts are inadmissible for a thin layer since experiment shows that an unstable layer at once breaks up into separate conical peaks whose dimensions, number, and spatial periodicity depend on the magnetic-field intensity. As the field intensity is increased each peak separates into two smaller ones.

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